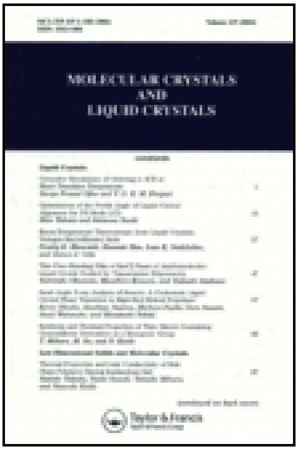

Molecular Crystals and Liquid Crystals Incorporating Nonlinear Optics
Publication details, including instructions for authors and subscription information:
http://www.tandfonline.com/loi/gmcl17

Electrohydrodynamic Instabilities in Nematic Liquid Crystals: Stability Criteria for Stochastic Excitation

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Version of record first published: 22 Sep 2006.

Mol. Cryst. Liq. Cryst. 1990, Vol. 193, pp. 235–239
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Printed in the United States of America

ELECTROHYDRODYNAMIC INSTABILITIES IN NEMATIC LIQUID CRYSTALS: STABILITY CRITERIA FOR STOCHASTIC EXCITATION

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Abstract Different stability criteria, as first moments stability, energetic stability, and sample stability are applied to a linear model for electrohydrodynamic instabilities in nematics. The influence of boundary conditions on instabilities is investigated comparing results for 1d and 2d theories.

THE MODEL

We consider electrohydrodynamic instabilities in nematic liquid crystals parametrically driven by a superposition of a deterministic electric field $\mathcal{E}_1$ with a stochastic dichotomous field $\mathcal{E}_t$ jumping with mean frequency α between $\pm\mathcal{E}$. Supposing that the transition from the nematic phase towards the Williams domain is continuous we investigate the linear model of Dubois-Violette et al.[1], which can be written as

$$\dot{\vec{z}} + (A + B\, \mathcal{E}_t)\, \vec{z} = 0 \, , \qquad (1)$$

where $\vec{z} = (q, \psi)$, q is the space charge density, ψ is the curvature of the angle between the director and the electrode plane. A and B are constant 2×2 matrices specific for 1d and 2d theories and depending on material parameters, on $\mathcal{E}_1$, and on the wave numbers k_x and k_z of the test mode.

MOMENT STABILITY

Since $\varepsilon_t^2 = \varepsilon^2$ = const it is possible to obtain closed equations for the moments of the form

$$(\mathbb{1}d/dt + C) (\langle \vec{w} \rangle, \langle \varepsilon_t \vec{w} \rangle)^T = 0 \quad . \qquad (2)$$

C depends on the same parameters as A and B, and on α. For the first moment , it is a 4×4 matrix and $\vec{w}=\vec{z}$, for the second moment it is a 6×6 matrix and $\vec{w}=(q^2, q\psi, \psi^2)$.

The exponential solutions of Eq. (2) are unstable if the largest exponent has a positive real part. The threshold value of the external field is obtained after minimizing the stable region with respect to the wave numbers (mode selection).

The stability analysis of the first moments for the 1d case[2] (no boundary conditions) can qualitatively explain the experimentally observed direct transition towards chaos above a critical strength of the stochastic field[3] (Figure 1).

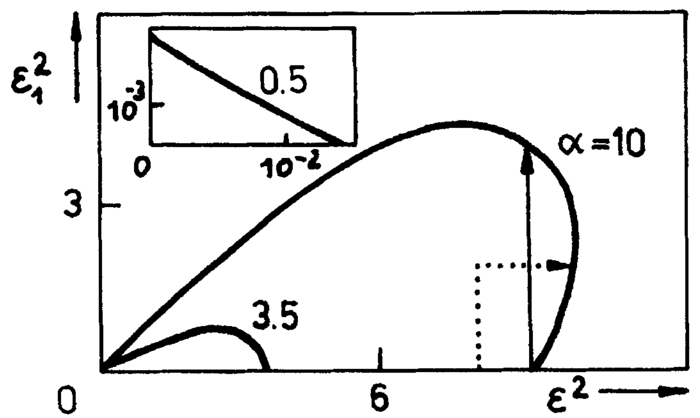

FIGURE 1 First moment's threshold curves for different α. Typical cases: Discontinuous behaviour for α=10 (solid arrow), the dashed part of the threshold is observable measuring along the dotted arrow; continuous behaviour (α=3.5 and α =0.5). For small α the noise is destabilizing. (Helfrichs parameter ζ^2=2.5, arb. units)

Since the quantitative agreement with the experiment is rather poor (the amplitude threshold is by far too high) we compare within the same frame different stability criteria.

The energetic stability criterion (i.e. the stability of the second moments) gives always a lower threshold with the same qualitative behaviour.

SAMPLE STABILITY

Introducing polar coordinates $(q,\psi) \rightarrow (r,\varphi)$ one obtains from Eq. (1)

$$\dot{r} = g(\varepsilon_t, \varphi)\, r \quad , \quad \dot{\varphi} = h(\varepsilon_t, \varphi) \quad , \qquad (3a,b)$$

where g and h are nonlinear functions of ε_t and φ, depending on the same parameters as A and B. Eq. (3a) is linear in r and can be solved for a given trajectory of the driving process. This leads to the Lyapunov exponent[4]

$$\lambda = \lim_{t\to\infty} \frac{1}{t} \int_0^t d\tau\; g(\varepsilon_\tau, \varphi) \;=\; \sum_{\sigma=\pm} \int dP_\sigma(\varphi)\; g(\sigma\varepsilon, \varphi) \;, \qquad (4)$$

the latter equality holds because of the multiplicative ergodic theorem of Oseledec. $P_\sigma(\varphi)$ is the invariant measure of the joint process (ε_t, φ) which can be obtained exactly from Eq. (3b). The threshold from the sample stability criterion is always higher than first moment's threshold.

Therefore, and in analogy with the stochastic parametric oscillator[5] we propose to prefer the energetic stability criterion.

In Figure 2 the thresholds corresponding to the above criteria are compared for the simplest case of pure stochastic excitation.

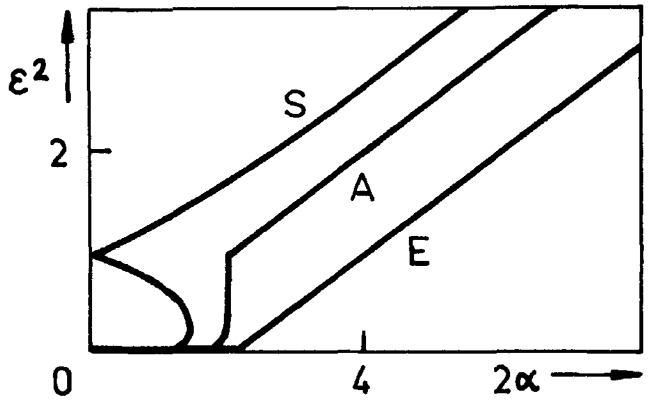

FIGURE 2 Pure stochastic excitation. Threshold corresponding to different criteria (sample (S), amplitude (A), and energetic (E) stability) against α ($\varsigma^2=3$).

INFLUENCE OF BOUNDARY CONDITIONS

Since in 1d theories no boundary conditions appear we investigated also a 2d linear theory with the simplification of free boundary conditions, The first moment's thresholds are slightly above those of the 1d

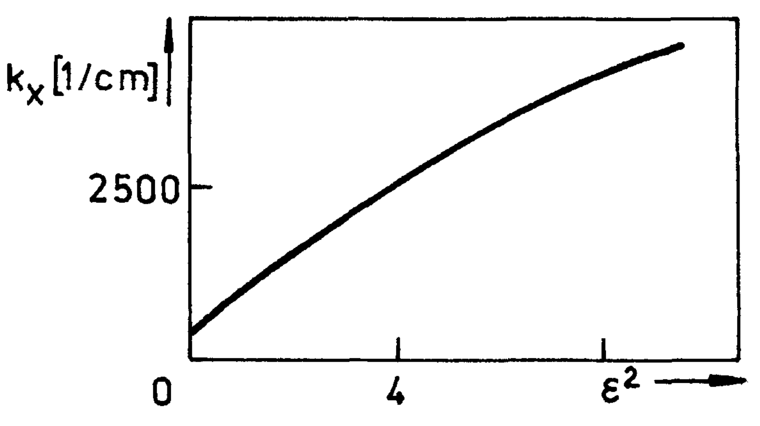

FIGURE 3 Wave number k for roll cells in a 2d theory against the stochastic part of the driving field ε^2 ($\varsigma^2=2.38$, $\alpha=8$).

theory but show the same qualitative behaviour (see Figure 1). In qualitative agreement with the experiment[3] we find that the wave number k_x of the roll cells increases with increasing strength of the stochastic field (Figure 3) as well as with increasing correlation time of the stochastic field (in the 1d model k_x is always constant).

CONCLUDING REMARKS

In all considered cases we are able to obtain results which are exact in the frame of the model. Especially, we avoid 'adiabatic' elimination procedures. More details will be published elsewhere[6].

To investigate the possibility of a discontinuous transition (which would be signaled by a hysteresis) one should treat a nonlinear model.

To explain the appearance of different patterns along the threshold curve[3] it would be necessary to test the stability of a 3d theory against the corresponding test modes.

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